The Michelson-Morley Experiment:

AN IMPROVED METHOD OF CALCULATING PHASE SHIFT ARISING IN THE MOVING INTERFEROMETER

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Abstract

By means of the Michelson interferometer a number of experiments were carried out to demonstrate the translational motion of the earth. The author re-analyzes the method of calculation in these experiments, and shows that, when taking into account a factor so far judged negligible, the expectable total phase shift will be proportional to the fourth *power of v/c (the ratio of translation speed to light velocity). No hypothesis relating to the existence of a physical phenomenon or law unknown to date is cited in this article.*

1. Introduction

More than a century has elapsed since 1887 when A. A. MICHELSON and E. W. MORLEY conducted their famous experiment [1], which was expected to reveal the translational motion of the earth. Although this experiment was repeated many times in the course of the years with the use of increasingly more sensitive equipment, no success could be attained in a reliable demonstration of the translational motion of earth. In addition to the experiments carried out with the use of the Michelson interferometer, experiments based on other methods have been conducted for the detection of this motion, but each of these also had a negative result or at least, a disputed one.

All these facts, however, do not exclude the possibility of making another analysis of the mentioned experiments on the basis of appropriate considerations. Of these, the author has selected the Michelson-Morley experiment as that to which a great conclusive force can be attributed, and, he now suggests an improved method of calculation.

Although the differences existing between the traditional and improved methods of calculation will be pointed out in several places, this article is not intended to give a comprehensive critique of the traditional method. The author's purpose is *merely* to draft a possible theoretical structure for calculation of arising phase shift.

It should be mentioned in advance that no hypothesis relating to the existence of a physical phenomenon or law unknown to date will be cited in this article.

We would like to emphasize that in our reasoning we shall adhere to the cornerstone of the traditional method of discussion, according to which light can be interpreted as a "classic" particle/wave phenomenon; i.e. light is a stream of corpuscles (photons) and at the same time it constitutes harmonic oscillations of electromagnetic field, therefore it propagates as a *ray* and also as a *sine wave*.
For this reason, the microstructure of light will not dealt with, and no quantum-optical considerations will be made in the calculation of the characteristics of interference. It is furthermore assumed that the characteristics of the experimental equipment and conditions have been idealized. Thus, for example, it is considered that light emitted from a source has a narrow spectrum and a large coherence length, etc.
In accordance with the traditional method of calculation the light diverging axial-symmetrically from the axes (arms) of the interferometer will not be drawn into the discussion; that is, the improved method of calculation will be applied only to the rays that are propagating *strictly* along the axes of the interferometer. In the figures also only these rays will be indicated.

As the Michelson-Morley experiment is well known, the related path calculation method will be described only in a concise summary in the Appendix, and the formulae and figure reported there will be referred to with the letter ***A*** preceding their reference numbers.

A scheme of the Michelson interferometer is shown in Fig. 1 in a position in which one of the arms - let this be K_2 - is parallel to the direction of motion.

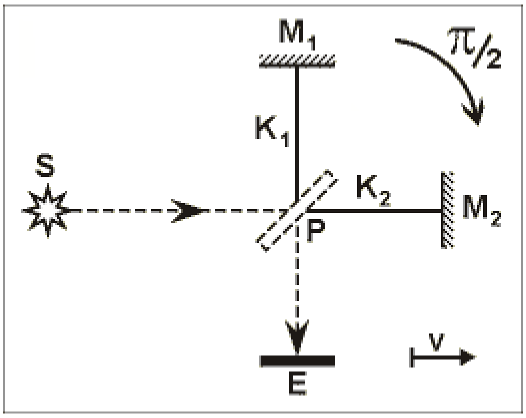

Figure 1. Scheme of the Michelson interferometer.

In the figure: **S** -- light source; **E** -- screen for displaying the interference pattern; $\mathbf{K_1}$, $\mathbf{K_2}$ -- perpendicular arms of the interferometer; $\mathbf{M_1}$, $\mathbf{M_2}$ -- mirrors mounted on the ends of the arms; **P** -- semi-transparent mirror for splitting and recombining the light source's beam; **v** -- velocity vector of the translation.

When the interferometer used in the experiment is rotated through angle $\pi/2$, and one of the arms is parallel to the direction of motion in the initial and final positions (hereafter called main positions), according to traditional method the value of total phase shift arising between the rays travelling along the arms can be expressed in radians as follows:

$$\Delta\Phi_{\Sigma} = 2\pi \frac{2l_1 + 2l_2}{\lambda} \left(\frac{1}{1\text{-}\beta^2} - \frac{1}{\sqrt{1-\beta^2}} \right) \cong k \frac{\beta^2}{2} \quad .$$

In the above formula:

$\Delta\Phi_{\Sigma}$	--	total phase shift arising in the two main positions of the interferometer,
π	=	3.14… (Ludolf's number),
l_1, l_2	--	length of the arms K_1 and K_2 of the interferometer,
λ	--	wavelength of light used for measurement,
β	=	v/c, where
v	--	speed of the interferometer's motion (speed of earth's translational motion),
c	--	propagation speed of light,
k	=	$2\pi(2l_1+2l_2)/\lambda$ (constant value for experimental apparatus).

According to the improved method of calculation suggested by the author, the total phase shift will be calculable with the use of the formula

$$\boxed{\Delta\Phi_{\Sigma} = 2\pi \frac{2l_1 + 2l_2}{\lambda} \left(\frac{1}{1\text{-}\beta^2} - \frac{2}{\sqrt{1-\beta^2}} + 1 \right) \cong k \frac{\beta^4}{4}} \quad , \tag{1}$$

that is, the arising overall fringe shift in interference pattern will be proportional to *fourth* order effect in v/c.

In the following, we shall discuss how the formula (1) can de derived.

2. Inaccuracy implied in the traditional method of calculation

The progression of the interfering rays in the moving interferometer is shown in Fig. 2. The paths of rays are depicted in accordance with the traditional model. Those rays that propagate along the arms here are illustrated and only the positions of mirrors M_1, M_2 and the semi-transparent mirror P are indicated.

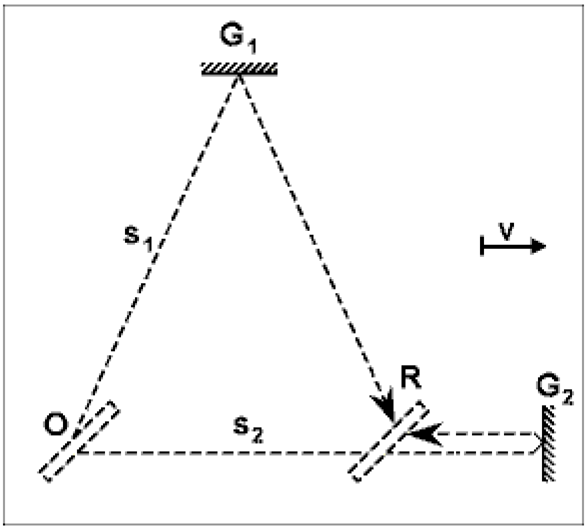

Figure 2. A model of light paths employed in the traditional method of calculation. According to this model, in the interferometer moving with translational speed v the light rays starting from the same position in the space will return to a common point.

In the figure: **O** -- position of semi-transparent mirror **P** at the start of the light rays; **R** – common return point of rays (position of mirror **P** on the return of the rays); $\mathbf{s_1}$, $\mathbf{s_2}$ -- paths of light rays along arms $\mathbf{K_1}$, $\mathbf{K_2}$, respectively; $\mathbf{G_1}$, $\mathbf{G_2}$ -- positions of mirrors $\mathbf{M_1}$ and $\mathbf{M_2}$ mounted on the ends of the arms upon the reflection of the light rays.

For the paths traversed, the following relationships can be written on the basis of formulae (***A***-5) and (***A***-6):

$$s_1 = \frac{2l_1}{\sqrt{1-\beta^2}} \quad ,$$

$$s_2 = \frac{2l_2}{1-\beta^2} \quad .$$

Thus, the sine waves of light arriving at point R from point O at moment t_R are described by the following equations:

$$a_1 = A\sin\omega\left(t_R - \frac{s_1}{c}\right) \quad ,$$

$$a_2 = A\sin\omega\left(t_R - \frac{s_2}{c}\right) \quad ,$$

where

a_1, a_2 -- instantaneous amplitude of light rays travelling along arms K_1 and K_2 respectively in point R and at moment t_R,

A -- amplitude of light wave,

ω -- cyclic frequency,

$\omega\left(t_R - \frac{s_1}{c}\right) = \Phi_1$ -- phase of the ray of arm K_1 in point R and at moment t_R,

$\omega\left(t_R - \frac{s_2}{c}\right) = \Phi_2$ -- phase of the ray of arm K_2 in point R and at moment t_R.

The rays emerge from a common light source, and for this reason the initial phases of the oscillations in the equations of the rays are identical. For the sake of simplicity, the initial phases have been taken for zero in the equations of the rays. This simplification will be employed later as well.

Subsequently, in addition to the commonly used concept of the initial phase the term "start phase of the ray" will be introduced as an auxiliary concept. For this reason it is considered very important that even now a clear discrimination should be made between the two concepts.

The term *initial phase* is used to denote the phase occurring at the spatial origin of the wave at moment t = 0. According to previous convention, this initial phase will be assumed as having a zero value.

The term *start phase* of the ray is used to denote the phase occurring at the spatial origin of the wave at an arbitrary moment t, and thus the notion of start phase appears in this paper as a generalization of the concept "initial phase". According to this, at the moment t = 0 the start phase value of the wave will be equal to the initial phase value, while at other moments it differs from that by ωt. Thus, the start phase can be regarded as a function of time.

In Fig. 2, the ray progressions shown in the model *tacitly* assume that the light rays starting from an the same (point O) of the moving mirror P and travelling strictly along the arms will return to a common location (point R).

Let us now show that this assumption in the model, as shown in Fig. 2, generally leads to an inaccuracy.

The light ray advancing along arm K_1 at speed c covers the path s_1 during a time t_1.

During this time interval mirror P travelling at velocity v traverses a path

$$vt_1 = v\frac{s_1}{c} = \beta s_1 \quad .$$

Likewise, the light ray advancing along arm K_2 at velocity c covers path s_2 during a time t_2. In this time interval, mirror P moving at velocity v covers a path of

$$vt_2 = v\frac{s_2}{c} = \beta s_2 \quad .$$

Taking this fact into consideration, the accurate ray path of the traditional model can generally be illustrated as shown in Fig. 3.

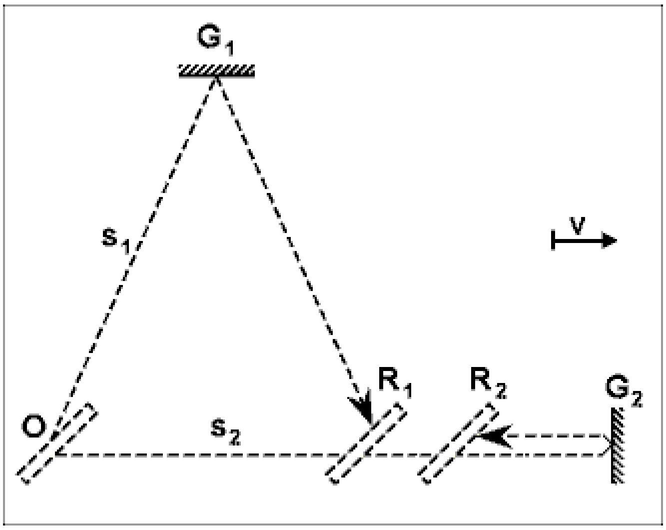

Figure 3. Analysis of the velocity and path conditions reveals that generally the return point of light rays starting from the same location can be different. This fact has already been pointed out by Michelson and Morley, but they decided its effect negligible.

In the figure: $\mathbf{R_1}$ -- return point of the ray of arm $\mathbf{K_1}$ to mirror **P**; $\mathbf{R_2}$ -- return point of the ray of arm $\mathbf{K_2}$ to mirror **P**; **others** -- see above.

According to the earlier analysis, the lengths of sections OR_1 and OR_2 are:

$$OR_1 = \beta s_1 \quad ,$$

$$OR_2 = \beta s_2 \quad .$$

In a given position of the interferometer the length of these two sections could be equal only when a relationship of

$$l_1 = \frac{l_2}{\sqrt{1-\beta^2}}$$

exists between the lengths of the arms. However, upon rotation of the interferometer, the equality of sections OR_1 and OR_2 ceases to exist even in the case of the existence of the aforementioned relationship. According to this, since the two sections are not generally equal, point R splits into points R_1 and R_2.
The question detailed above was also mentioned [2] in article [1]. However, in the absence of any de-tailed numerical assessment thereof, its effect was considered negligible.

The relevant literature seems to be uniform in discussing the principles of method of calculation applied with the Michelson interferometer. It ignores the apparent possibility of inaccuracy, or rather, it does not even make mention of it. For this reason, the references are limited to article [1], since it has already revealed the possible source of inaccuracy.

The problem becomes particularly acute, if the lengths of the arms (l_1, l_2) differ greatly from each other. Then, the lengths of light paths (s_1, s_2) may also be considerably dissimilar with each other. Consequently, the distance

$$R_2R_1 = \beta\,(s_2 - s_1)$$

existing between the returning light rays could also be increased significantly. In such a case, if a point-like test object were placed between the light source and mirror P, a doubling of the image of the test object, in compliance with the traditional model, would become possible upon sufficient change in translation velocity.

To summarize, it seems to be expedient to make a proposal for a model that excludes the previously revealed possibility of inaccuracy, or one that can be employed in the case of an arbitrary arm length and velocity.

3. The proposed model and method of calculation

Accurately retaining the path length of the rays (s_1, s_2), we must correct the traditional model so that the above mentioned possibility of inaccuracy will not occur in the modified model. The proposed model, shown below in Fig. 4, attempts to fulfill these requirements.

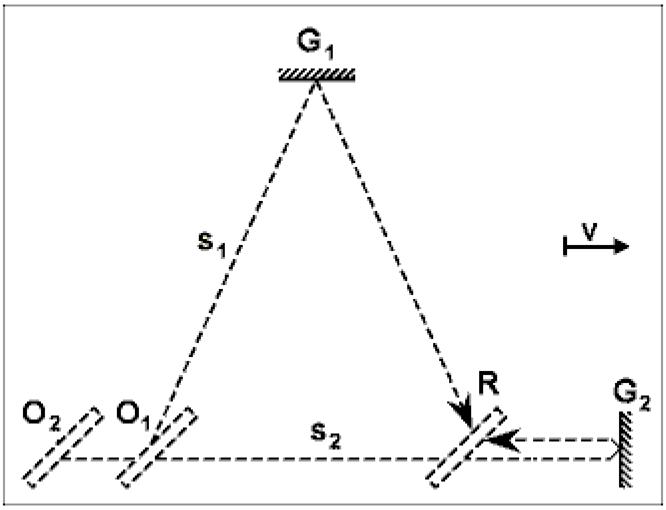

Figure 4. Model of the light paths applied to the improved method of calculation. The corrected model permits rays returning to a common point to start from different points in space. At the same time this also implies that the *start phases* of the two light rays can differ from each other. The possibility of inaccuracy inherent in the traditional method can be eliminated by using this model.

In the figure: **R** -- common return point of rays at mirror **P**; $\mathbf{O_1}$ -- starting point of a ray of arm $\mathbf{K_1}$ from mirror **P**; $\mathbf{O_2}$ -- starting point of a ray of arm $\mathbf{K_2}$ from mirror **P**; **others** -- see above.

While in the traditional model (shown in Fig. 3) the rays propagating strictly along the axes started from the same point O but arrived at different spatial points R_1 and R_2, these rays return to the same point R in the proposed corrected model. As a consequence of this, the previously mentioned possibility of inaccuracy cannot occur regardless of the length of the arms or the speed of the translational motion.

In the revisited model, however, it has to be taken into account that the rays returning to point R may start from different positions of the moving mirror P (points O_1 and O_2) and, consequently, at different moments in time. Therefore, the *start phase* of these rays can differ by some $\omega\Delta t$, where Δt is the difference between the starting times of the rays outgoing from points O_1 and O_2.
Thus, not only is the change in spatial conditions (path lengths) taken into account among the consequences of motion, but the traditional method of calculation is enhanced by regarding the altered time conditions (start phases) as well.

The same evident procedure employed by the traditional method will be used to calculate the path lengths of the light rays. The method of calculation in this article will be improved *only* by determining the difference of start phases $\omega\Delta t$ of the light rays outgoing from points O_1 and O_2. Here, ω is constant and known and thus the unknown Δt has to be calculated.
In the course of the calculations it should be naturally requested that in the case of $v \to 0$ each of the formulae should be transformed into generally accepted formulae or should adopt those values, that this formulae give for resting interferometers. The reader can check the fulfillment of this requirement later.
The interferometer will be broken down into two subsystems, one each for arms K_1 and K_2, for the calculations. The quantities of the individual subsystems will, as previously, be distinguished by subscripts 1 and 2. To unite the two subsystems the time moment t_R, when the arms just pass through the point R, was chosen as the common reference point for the time-counting.

In what follows, the path of a ray in one of the subsystems, let this be the arm K_1 subsystem, will be studied in detail. To this end, a ray that travels along the appropriate axis in an arbitrary position of the interferometer, will be plotted on the basis of Fig. ***A***-1. In this position the arm K_1 subtends angle α with the direction of motion, and the characteristics of light ray can readily be examined.
The path length s_1 and deflection angle γ_1 of the path of the ray shown in the figure are yielded by the calculations contained in the Appendix (as a function of quantities l_1, α and β).

To facilitate the analysis, the light source is assumed to be located directly at mirror P which is rigidly connected with the arms. Fig. 5 shows the equivalent image of the light source, which can be observed at point R. This image is considered a virtual light source.

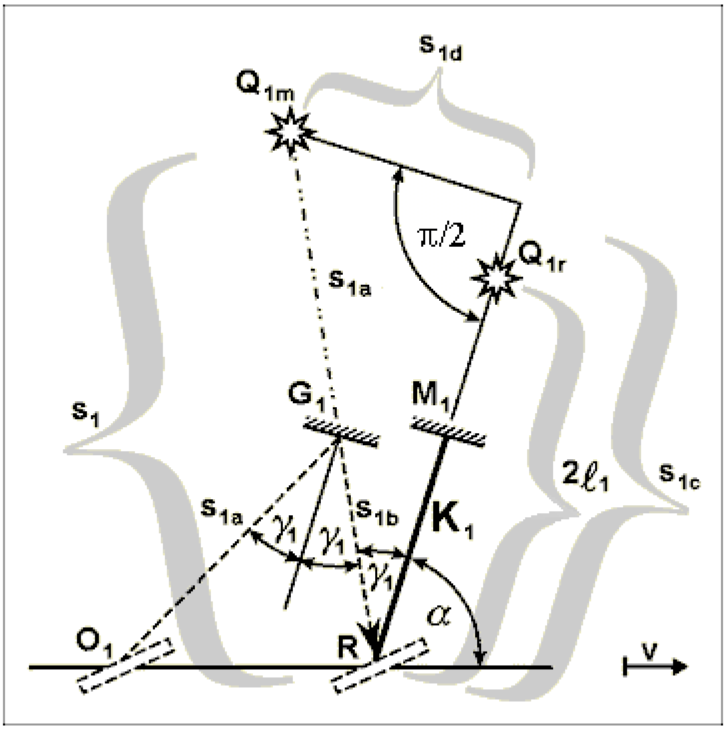

Figure 5. Path of a light ray travelling along arm K_1 of the interferometer in case of an arbitrary angle α. In order to achieve a surveyable illustration and facilitate the calculations, the path of a ray is shown as outgoing from the virtual light source, which can be observed at point R. With the interferometer at rest, a light ray would travel a path of $2l_1$ in length along the arm. When the interferometer is moving, the path of light becomes elongated ($s_1 > 2l_1$), and it subtends angle γ_1 with the arm.

In the figure: $\mathbf{Q_{1r}}$ -- virtual light source visible from point **R** with the interferometer at rest; $\mathbf{Q_{1m}}$ -- virtual light source visible from point **R** with the interferometer in motion; $\boldsymbol{\gamma_1}$ -- angle subtended by the light ray with arm $\mathbf{K_1}$; $\mathbf{s_{1a}}$, $\mathbf{s_{1b}}$ -- portions of the path of a light ray from point $\mathbf{O_1}$ to point $\mathbf{G_1}$ and from point $\mathbf{G_1}$ to point **R**, respectively; $\mathbf{s_{1c}}$, $\mathbf{s_{1d}}$ -- components of the path of light ray $\mathbf{s_1}$ parallel and perpendicular respectively to arm $\mathbf{K_1}$; **others** -- see above.

We shall investigate the position of the virtual light source in two different states of motion. In the figure Q_{1r} denotes the position of virtual light source in a resting interferometer ($v = 0$), while Q_{1m} marks the position of virtual light source in a moving interferometer ($v > 0$). For a better demonstration of the spatial positions Q_{1r} and Q_{1m} of virtual light source, a prolonged line is drawn along arm K_1 with a short break in continuity.

With the interferometer at rest, the virtual light source would constantly be located in point Q_{1r} on the prolonged line of arm K_1, i.e. at moment t_R it can also be found there. At this time the light ray starting from point Q_{1r} could be described in point R and at moment t_R by the equation

$$a_1 = A \sin \omega \left(t_R - \frac{2l_1}{c} \right) \quad .$$

When the interferometer is moving at speed v, the light ray in point R and at moment t_R is not represented correctly by the above equation because

-- the distance to be covered by the ray from the moving virtual light source Q_{1m} is lengthened and becomes not $2l_1$ but longer by some Δl_1 value, i.e. $s_1 > 2l_1$;

-- the direction of the ray arriving in point R is also changed; starting now from point Q_{1m} instead of Q_{1r}, the ray subtends angle γ_1 with arm K_1 during its propagation, i.e. $\gamma_1 > 0$.

To sum up, it may be stated that the path of the light ray not only lengthened ($s_1 > 2l_1$) but also "turned back" from arm K_1 around point R ($\gamma_1 > 0$), and, thus, the light source Q_{1m} has arrived behind the prolonged line of arm K_1.

As we previously assumed, the light source is located at mirror P which is rigidly connected with the arms, and, the arms pass through point R at moment t_R. Therefore, the virtual light source had to cross point Q_{1m} in the process of motion at a moment *prior* to t_R. This moment is denoted by $t_R - \Delta t_1$, where Δt_1 is a still unknown time interval.

Consequently, the start phase of the light ray outgoing from point Q_{1m} is not ωt_R but something smaller by an $\omega\Delta t_1$ value.

According to the above reasoning, in the case of a moving interferometer the light ray is described correctly in point R and at moment t_R by the equation

$$a_1 = A\sin\omega\left(t_R - \Delta t_1 - \frac{2l_1 + \Delta l_1}{c}\right) \quad .$$

This formula reflects the fact that in point R the phase of the ray is affected by the lengthened path (Δl_1) as well as the changed start phase ($\omega\Delta t_1$) on account of the light ray's deflection.

Hence, the total length of the light's path will be

$$s_1 = 2l_1 + \Delta l_1 \quad .$$

Since the value of s_1 is already familiar from formula (***A***-1), Δl_1 will not be discussed separately. Accordingly, for further calculations it may be written that

$$a_1 = A\sin\omega\left(t_R - \Delta t_1 - \frac{s_1}{c}\right) \quad . \tag{2}$$

Let us now determine the value of the unknown time interval Δt_1 .

To this end, with reference to Fig. 5, let us break path s_1 into two components:

-- component s_{1c} coinciding with the direction of arm K_1, and

-- component s_{1d} dropped perpendicularly from point Q_{1m} on prolonged line of arm K_1. The appearance of this component is based on a deflection of the light ray and its dimension is defined by the distance between the virtual light source Q_{1m} and the prolonged line of arm K_1.

When the value of component s_{1d} is zero, i.e. when the virtual source is located on the line of arm K_1, the value Δt_1 obviously will also be zero. (This also holds true when simultaneously $s_{1c} > 2l_1$, which is possible if $\alpha = 0$ and $v > 0$.) On the basis of this, it can be stated that the appearance of the time interval Δt_1 is due to the spatial distance s_{1d} arising between the virtual light source Q_{1m} and the line of arm K_1.

The light ray covers the path s_1 during a time

$$t_1 = \frac{s_1}{c} \quad .$$

Should the light travel along s_{1c}, time interval t_{1c} would require:

$$t_{1c} = \frac{s_{1c}}{c} \quad .$$

An unknown supplementary interval Δt_1 must be added to time interval t_{1c} to obtain time interval t_1, during which the light ray can cover path s_1. Thus, with interval Δt_1 added, the component s_{1d}, which is perpendicular to the arm, is created in addition to the component s_{1c} coinciding with the direction of the arm:

$$t_1 = t_{1c} + \Delta t_1 \quad .$$

According to the geometry of Fig. 5, the length of s_{1c} is

$$s_{1c} = s_1 \cos\gamma_1 \quad .$$

With t_1, t_{1c} and s_{1c} eliminated, the unknown Δt_1 can be obtained from the above set of four equations as a function of quantities s_1 and γ_1.

Solving the set of equations will yield

$$\Delta t_1 = \frac{s_1}{c}\left(1 - \cos\gamma_1\right) \quad . \tag{3}$$

On the basis of relationship (***A***-2), this may also be written as

$$\Delta t_1 = \frac{s_1}{c}\left(1 - \sqrt{1 - \beta^2 \sin^2\alpha}\right) \quad .$$

Notably, in the case of $\alpha = 0$ (arm K_1 is parallel to the direction of motion) Δt_1 will assume a zero value at any velocity, since the light ray does not deflect laterally from the arm.

An entirely analogous procedure can be adopted in investigation of a ray travelling in the arm K_2 subsystem. Thus,

$$a_2 = A \sin\omega\left(t_R - \Delta t_2 - \frac{s_2}{c}\right) \quad , \tag{4}$$

where

$$\Delta t_2 = \frac{s_2}{c}\left(1 - \cos\gamma_2\right) \quad . \tag{5}$$

As previously, let us introduce the notations

$$\omega\left(t_R - \Delta t_1 - \frac{s_1}{c}\right) = \Phi_1 \quad ,$$

$$\omega\left(t_R - \Delta t_2 - \frac{s_2}{c}\right) = \Phi_2$$

in the formulae (2) and (4), where Φ_1 and Φ_2 are the phases of light rays returning to point R at moment t_R. Let us determine the phase shift $\Delta\Phi$ of the two incident rays for an arbitrary angle α .

$$\Delta\Phi = \Phi_1 - \Phi_2 = \omega\left(t_R - \Delta t_1 - \frac{s_1}{c}\right) - \omega\left(t_R - \Delta t_2 - \frac{s_2}{c}\right) = \omega\left[\frac{s_2 - s_1}{c} - \left(\Delta t_1 - \Delta t_2\right)\right] \quad ,$$

where Δt_1 - Δt_2 = Δt gives the difference between the starting times of the rays outgoing from points O_1 and O_2.

Let us calculate the total phase shift $\Delta\Phi_\Sigma$ for main positions, when arm K_1 was at a right angle to the direction of motion ($\alpha = \pi/2$) before the interferometer was rotated through angle $\pi/2$. Then

$$\Delta\Phi_\Sigma = \Delta\Phi\,|_{\alpha=\pi/2} - \Delta\Phi\,|_{\alpha=0} = \omega\left[\frac{s_2 - s_1}{c} - \left(\Delta t_1 - \Delta t_2\right)\right]\Bigg|_{\alpha=0}^{\alpha=\pi/2} \quad .$$

By substituting here the values of s_1 and s_2 from expressions (***A***-1) and (***A***-3), Δt_1 and Δt_2 from expressions (3) and (5), cos γ_1 and cos γ_2 from expressions (***A***-2) and (***A***-4), and by using relationship

$$\frac{\omega}{c} = \frac{2\pi}{\lambda} \quad ,$$

the following equation may be written:

$$\Delta\Phi_\Sigma = 2\pi \frac{2l_1 + 2l_2}{\lambda}\left(\frac{1}{1-\beta^2} - \frac{2}{\sqrt{1-\beta^2}} + 1\right) \quad . \tag{6}$$

Assuming that $\beta \ll 1$, let us expand the expressions $1/\left(1-\beta^2\right)$ and $2/\sqrt{1-\beta^2}$ into a series, omitting the higher-order members from β^6 onward. Then the following expression will be obtained:

$$\frac{1}{1-\beta^2} - \frac{2}{\sqrt{1-\beta^2}} + 1 = 1 + \beta^2 + \beta^4 + \ldots - 2\left(1 + \frac{1}{2}\beta^2 + \frac{3}{8}\beta^4 + \ldots\right) + 1 \cong \frac{\beta^4}{4} \quad . \tag{7}$$

By using (7) and the earlier introduced notation $k = 2\pi(2l_1+2l_2)/\lambda$, formula (6), which supplies the value of the total phase shift $\Delta\Phi_\Sigma$, may be written as follows:

$$\Delta\Phi_\Sigma \cong k\frac{\beta^4}{4} \quad .$$

Thus, formula (1), which was presented earlier in this paper, has been completely deduced.

APPENDIX

Path calculation of the light rays

Let us examine the paths of rays that travel strictly along the arms, with the interferometer in an arbitrary position, when arm K_1 subtends angle α with the direction of motion. The interferometer is moving at the rate of speed of v, and the arms K_1 and K_2 have a length of l_1 and l_2, respectively.

The path of a ray of arm K_1 is indicated in Fig. ***A***-1. For simplification of the formulae, let us stipulate that $0 \le \alpha \le \pi/2$.

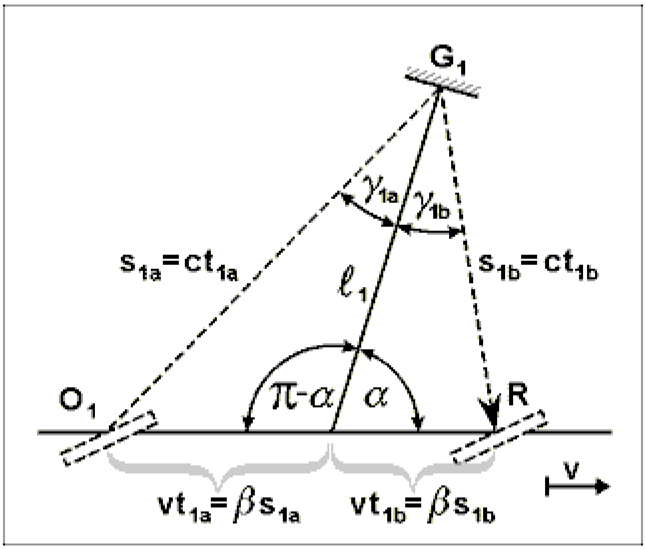

Figure *A*-1. The path of a ray of arm K_1 in the interferometer moving with translational speed v. Arm K_1 subtends an arbitrary angle α with the direction of motion.

In the figure: $\mathbf{O_1}$ -- position of semi-transparent mirror **P** at the start of the light ray; **R** -- return point of light ray to mirror **P**; $\mathbf{G_1}$ -- position of mirror $\mathbf{M_1}$ mounted on the end of arm upon the reflection of the light ray; $\mathbf{s_{1a}}$, $\mathbf{s_{1b}}$ -- portions of the path of a light ray from point $\mathbf{O_1}$ to point $\mathbf{G_1}$ and from point $\mathbf{G_1}$ to point **R**, respectively; $\boldsymbol{\gamma_{1a}}$, $\boldsymbol{\gamma_{1b}}$ -- angles subtended by a light ray with arm; $\mathbf{t_{1a}}$ -- time interval in which the ray of light arrives at point $\mathbf{G_1}$ from point $\mathbf{O_1}$; $\mathbf{t_{1b}}$ -- time interval in which the light ray arrives at point **R** from point $\mathbf{G_1}$.

Let us determine the quantities s_{1a}, s_{1b}, γ_{1a} and γ_{1b} (giving the path lengths and inclination angles of the ray) as the functions of quantities l_1, α and β.

Determination of the quantities sought is a simple geometric task; however, for the sake of completeness, the calculations are given below.

With reference to the figure, on the basis of the cosine law, it may be written that

$$s_{1a}^2 = l_1^2 + (\beta s_{1a})^2 + 2 l_1 \beta s_{1a} \cos(\pi - \alpha) \quad ,$$

$$s_{1b}^2 = l_1^2 + (\beta s_{1b})^2 + 2 l_1 \beta s_{1b} \cos\alpha \quad .$$

From the foregoing, the total length of the path of a ray from point O_1 to point R is

$$s_1 = s_{1a} + s_{1b} = \frac{2l_1}{1-\beta^2}\sqrt{1-\beta^2\sin^2\alpha} \quad . \qquad (\boldsymbol{A}\text{-}1)$$

On the basis of the sine law, it may be written that

$$\frac{\sin\gamma_{1a}}{\sin(\pi-\alpha)} = \frac{\beta s_{1a}}{s_{1a}} = \beta \quad ,$$

$$\frac{\sin\gamma_{1b}}{\sin\alpha} = \frac{\beta s_{1b}}{s_{1b}} = \beta \quad .$$

From the above expressions

$$\sin\gamma_{1a} = \sin\gamma_{1b} = \beta\sin\alpha \quad ,$$

consequently,

$$\gamma_{1a} = \gamma_{1b} \quad ,$$

and, in what follows, γ_1 will be written instead of γ_{1a} and γ_{1b}. Hence,

$$\cos\gamma_1 = \sqrt{1-\sin^2\gamma_1} = \sqrt{1-\beta^2\sin^2\alpha} \quad . \qquad (\boldsymbol{A}\text{-}2)$$

The path of a ray can be calculated in a perfectly analogous way for arm K_2. The only difference is that arm K_2 subtends angle α - $\pi/2$, not α, with the direction of motion. Accordingly,

$$s_2 = \frac{2l_2}{1-\beta^2}\sqrt{1-\beta^2\cos^2\alpha} \quad , \qquad (\boldsymbol{A}\text{-}3)$$

$$\cos\gamma_2 = \sqrt{1-\beta^2\cos^2\alpha} \quad . \qquad (\boldsymbol{A}\text{-}4)$$

By substitution $\alpha = \pi/2$ into (***A***-1) and (***A***-3), the length of the paths will be obtained for the interferometer in the position shown in Fig. 1:

$$s_1 = \frac{2l_1}{\sqrt{1-\beta^2}} \quad , \qquad (\boldsymbol{A}\text{-}5)$$

$$s_2 = \frac{2l_2}{1-\beta^2} \quad . \qquad (\boldsymbol{A}\text{-}6)$$

References

[1] A. A. MICHELSON, E. W. MORLEY: On the Relative Motion of the Earth and the Luminiferous Ether
Amer. J. Sci., Vol. 134, No. 203, Nov. 1887, pp. 333 - 345.
See also a facsimile of the paper at https://history.aip.org/exhibits/gap/PDF/michelson.pdf

[2] "It may be remarked that rays ba_l and ca_l, do not now meet exactly in the same point a_l, though the difference is of the second order; this does not affect the validity of the reasoning."

Symbols used in the paper

Symbols beginning with Roman characters:

A	--	amplitude of light wave,
a_1, a_2	--	instantaneous value of amplitude of light ray travelling along arms K_1, K_2, respectively,
c	--	propagation velocity of light,
E	--	screen displaying the interference pattern,
G_1, G_2	--	position of mirror M_1 and M_2 on reflecting the light ray, respectively,
k	=	$2\pi(2l_1+2l_2)/\lambda$ (constant value for experimental apparatus),
K_1, K_2	--	arms of the interferometer,
l_1, l_2	--	length of arm K_1 and K_2 of the interferometer, respectively,
M_1, M_2	--	mirror mounted on the end of arm K_1, K_2, respectively,
O, O_1, O_2	--	position of semi-transparent mirror P on starting the light rays of arm K_1, K_2, respectively,
P	--	semi-transparent mirror for splitting and recombining the beam of the light source,
Q_{1m}, Q_{1r}	--	virtual source of light visible from point R in the moving and resting interferometer, respectively,
R	--	return point of the light rays at semi-transparent mirror P,
R_1, R_2	--	return point of the ray of arm K_1 and K_2 to semi-transparent mirror P, respectively,
S	--	light source,
s_1, s_2	--	total path of light ray travelling along arm K_1, K_2, respectively,
s_{1a}, s_{1b}	--	portion of the path of light ray from point O_1 to mirror G_1 and from mirror G_1 to point R, respectively,
s_{1c}, s_{1d}	--	component of path of light ray s_1 parallel to arm K_1 and at right angle to it, respectively,
t	--	time,
t_R	--	time moment when semi-transparent mirror P is in point R,
t_1	--	time interval in which the light ray covers path s_1,
t_{1a}, t_{1b}	--	time interval in which the light ray arrives from point O_1 at mirror G_1 and from mirror G_1 at point R, respectively,
t_{1c}	--	time interval in which the light ray would cover path s_{1c},
v	--	velocity vector of translational motion.

Symbols beginning with Greek characters:

α	--	angle subtended by arm K_1 with the direction of motion,
β	=	v/c,
$\gamma_1 = \gamma_{1a} = \gamma_{1b}$, γ_2	--	angle subtended by the light ray with the arm K_1, K_2, respectively,

Δl_1 -- increment of path of light $2l_1$ as a result of motion,

Δt -- difference between the start time instants of light rays outgoing from points O_1 and O_2,

Δt_1, Δt_2 -- time component based on path component at right angle to arm K_1, K_2, respectively,

$\Delta\Phi$ -- phase shift of the light rays,

$\Delta\Phi_\Sigma$ -- total phase shift in main positions of the interferometer,

λ -- wavelength of light used in the measurement,

π = 3.14... (Ludolf's number),

Φ_1, Φ_2 -- phase of the light ray of arm K_1, K_2, respectively,

ω -- cyclical frequency.

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